Orthogonal Time Frequency Space (OTFS) modulation for millimeter-wave communications systems

R. Hadani,[1] S. Rakib,[1] A. F. Molisch,[2] C. Ibars,[1] A. Monk,[1]
M. Tsatsanis,[1] J. Delfeld,[1] A. Goldsmith,[3] R. Calderbank, [4]

[1] Cohere Technologies, Santa Clara, CA; [2] University of Southern California, Los Angeles, CA,
[3] Stanford University, Stanford, CA, [4] Duke University, Durham, NC

***Abstract*—Due to the increased demand for data rate, flexibility, and reliability of 5G cellular systems, new modulation formats need to be considered. A recently proposed scheme, Orthogonal Time Frequency Space (OTFS), offers various advantages in particular in environments with high frequency dispersion. Such environments are encountered, e.g, in mm-wave systems, both due to the higher phase noise, and the larger Doppler spreads encountered there. The current paper provides a performance evaluation of OTFS at 5G mm-wave frequencies. Comparisons with OFDM modulation show that OTFS has lower BER than OFDM in a number of situations.**

***Index Terms*—5G mobile communication, mm-wave, time-frequency diversity, phase noise.**

I. Introduction

Demand for wireless services is constantly increasing, and a variety of new services, such as interactive gaming, 4k video streaming, smart homes, etc., are emerging. They will require greater flexibility, as well as higher reliability and higher data rate than the existing LTE (3GPP Long Term Evolution) and other 4G standards [1]. For this reason, 3GPP and other international standardization organizations have been developing the foundations for 5G systems; among other innovations, new modulation formats are being considered and evaluated.

A critically important component of 5G systems will be the use of millimeter-wave frequency bands, as they offer much larger bandwidth than the classical cellular bands. The higher free-space pathloss at those frequencies, which had long been considered a showstopper for cellular applications, can be compensated by beamforming gain of suitable adaptive arrays [2], [3], and many theoretical investigations as well as prototypes have shown the basic feasibility of realizing 5G systems at mm-wave frequencies [4], [5]. However, a major remaining challenge is the behavior in highly frequency-dispersive channels: since the phase noise and the sensitivity to movement (Doppler spread) increases with carrier frequency, finding modulation methods whose performance does not degrade significantly under those circumstances is critical.

We recently proposed a new modulation approach called OTFS (Orthogonal Time Frequency Space) modulation [6], [7]. It spreads the basis waveform (i.e., the signal multiplying each modulation symbol) over the whole time-frequency plane, in contrast to OFDM (orthogonal frequency division multiplexing) [8], where the basis waveform is highly localized. As a consequence, OTFS provides a high diversity order, and thus performs well particularly in situation with high Doppler spread and limited or no channel state information at the transmitter (CSIT). However, it can be conjectured that at extreme-valued Doppler spreads, as well as in the presence of phase noise, the ability of the receiver to cope with the dispersion will be limited, and resulting self-interference would degrade performance. Such situations might particularly occur in mm-wave bands. Since these scenarios have not yet been investigated, the current paper aims to close this gap.

The remainder of the paper is organized as follows: Section II reviews the fundamental principles of OTFS. Section III describes the simulation settings, including channel model and phase noise model. Section IV gives the performance results.

II. OTFS

OTFS modulation fundamentally operates in the delay-Doppler domain (essentially the 2D Fourier dual of the time-frequency domain), effectively converting the fading, time-variant channel experienced by OFDM into a non-fading, time-independent channel. Let the received signal $r(t)$ due to a transmit signal $s(t)$ be given by

$$r(t) = \iint h(\tau,\nu) e^{j2\pi\nu(t-\tau)} s(t-\tau)\, \mathrm{d}\nu\, \mathrm{d}\tau \qquad (1)$$

where $h(\tau,\nu)$ is the spreading function of the channel, i.e., the Fourier transform (with respect to t) of the time-variant impulse response $h(\tau,t)$ [8]. Here, τ and ν are delay and Doppler shift, respectively. The mapping of the transmit onto the receive signal can be interpreted as a Heisenberg transform, parameterized by the function $h(\tau,\nu)$.

The OTFS modulation itself can now be interpreted as a cascade of two two-dimensional transforms. At the transmitter, in a first step, the information symbols $x[n,m]$, which reside in the delay-Doppler domain, are mapped into the time-frequency domain through the 2D inverse symplectic Fourier transform. This mapping also involves windowing (since any practical implementation has finite block size) and periodization (indicated by subscript p for the corresponding quantities) with period (N,M): $X_p[n,m] = \mathrm{SFFT}^{-1}(x_p[k,l])$ for

$$X_p[n,m] = \frac{1}{MN} \sum_{l,k} x_p[k,l] e^{j2\pi(\frac{nk}{N} - \frac{ml}{M})}, \qquad (2)$$

where $l = 0,\dots,M-1, k = 0,\dots,N-1$. At the receiver, the inverse process uses the SFFT (symplectic finite Fourier

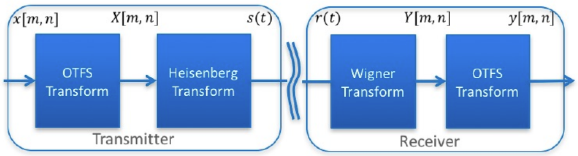

Fig. 1. OTFS Modulation Block Diagram: Transmitter and Receiver

transform) of $X[n,m]$ to obtain $x_p(k,l) = \text{SFFT}(X_p[n,m])$ for

$$x_p[k,l] = \sum_{n=0}^{N-1} \sum_{m=-\frac{M}{2}}^{\frac{M}{2}-1} X_p[n,m] e^{-j2\pi(\frac{nk}{N} - \frac{ml}{M})} \tag{3}$$

In the following, the combination of windowing and inverse SFFT is called the OTFS transform.

Subsequently, the time-frequency signal is converted into a time-domain signal $s(t)$ through a Heisenberg transform

$$s(t) = \sum_{m=M/2}^{M/2-1} \sum_{n=0}^{N-1} X[n,m] g_{\text{tx}}(t-nT) e^{j2\pi m\Delta f(t-nT)}. \tag{4}$$

where $X = W_{tx} * SFFT^{-1}(x_p)$ where W_{tx} is the time-frequency windowing function at the transmitter. This can be interpreted as a Heisenberg operator with parameters $X[n,m]$ applied to the "basis pulse" of the transmission $g_{\text{tx}}(t)$ (e.g., a rectangular pulse). For later use, note that the transmit and receive basis pulses fulfill the bi-orthogonality condition

$$\int g^*_{\text{tx}}(t) g_{\text{rx}}(t-nT) e^{j2\pi m\Delta f(t-nT)} \text{d}t = \delta(m)\delta(n). \tag{5}$$

One can thus interpret the received signal as a cascade of two Heisenberg operators working on the basis pulse, one describing the modulation (without the OTFS transform), and one describing the interaction with the channel. It is now known that the cascade of two Heisenberg operators with parameterizing functions h_1 and h_2 is another Heisenberg operator, parameterized with a function h that is the *twisted convolution* of the two original operators

$$\begin{aligned} &h_2(\tau,\nu) *_\sigma h_1(\tau,\nu) = \\ &\iint h_2(\tau',\nu') h_1(\tau-\tau',\nu-\nu') e^{j2\pi\nu'(\tau-\tau')} \text{d}\tau' \text{d}\nu'. \end{aligned} \tag{6}$$

Thus, the received signal becomes (disregarding the noise for ease of notation)

$$r(t) = \iint f(\tau,\nu) e^{j2\pi\nu(t-\tau)} g_{\text{tx}}(t-\tau) \text{d}\nu \text{d}\tau, \tag{7}$$

where $f(\tau,\nu)$, the impulse response of the combined transform:

$$\begin{aligned} &f(\tau,\nu) = h_{(}\tau,\nu) *_\sigma X[n,m] = \\ &\sum_{m=-.5M}^{.5M-1} \sum_{n=0}^{N-1} X[n,m] h(\tau-nT,\nu-m\Delta f) e^{j2\pi(\nu-m\Delta f)nT} \end{aligned} \tag{8}$$

The receiver performs a cascade of a Wigner transform and an OTFS transform. In other words, the signal is first filtered with the receive basis pulse. The delay-Doppler representation of this filtered signal can be interpreted as the cross-ambiguity function of the received signal with the basis pulse

$$A_{g_{\text{rx}},r}(\tau,\nu) \triangleq \int e^{-j2\pi\nu(t-\tau)} g^*_{\text{rx}}(t-\tau) r(t) \text{d}t. \tag{9}$$

sampled at $\tau = nT$ and at $\nu = m\Delta f$. It can be shown that this signal is

$$A_{g_{\text{rx}},\Pi_f(g_{\text{tx}})}(\tau,\nu) = f(\tau,\nu) *_\sigma A_{g_{\text{rx}},g_{\text{tx}}}(\tau,\nu) \tag{10}$$

and the end-to-end channel (without OTFS transform) can be described as

$$Y(\tau,\nu) = h(\tau,\nu) *_\sigma X[n,m] *_\sigma A_{g_r,g_{tr}}(\tau,\nu) \quad . \tag{11}$$

The receiver then performs an SFFT on the sampled, windowed, and periodized version of Y to obtain estimates of the received signal. The estimated sequence $\hat{x}[k,l]$ of information symbols obtained after demodulation is given by the two dimensional periodic convolution

$$\hat{x}[k,l] = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x[n,m] h_w\left(\frac{k-n}{NT}, \frac{l-m}{M\Delta f}\right) \tag{12}$$

of the input QAM sequence $x[n,m]$ and a sampled version of the windowed impulse response $h_w(\cdot)$,

$$h_w\left(\frac{k-n}{NT}, \frac{l-m}{M\Delta f}\right) = h_w(\nu',\tau')|_{\nu'=\frac{k-n}{NT},\tau'=\frac{l-m}{M\Delta f}} \tag{13}$$

where $h_w(\nu',\tau')$ denotes the circular convolution of the channel response with a windowing function, which in turn is the symplectic Fourier transform of the product of the transmit and receive window. A two-dimensional equalization, e.g., using a two-dimensional decision feedback equalizer, provides the final estimates of the information symbols.

It is noteworthy that each received symbol "sees" the same channel gain $h_w(0,0)$ on the transmitted symbol $x[l,k]$, and obtains full diversity from both the delay and the Doppler domain.

III. Performance Evaluation

A. System parameters and channel model

The parameters of the system we model here are in line with the setup considered by 3GPP, the standardization body for 5G cellular communications. We compare the OTFS system to an OFDM system, which is the state of the art for 4G. We consider subcarrier spacings of 15 kHz (used for current systems), as well as 60 kHz and 480 kHz, all of which are currently considered for mm-wave systems in 3GPP. For 15 kHz, a cyclic prefix length of $4.7\mu s$ is used (as in LTE). For higher sub-carrier spacings, the cyclic prefix is scaled accordingly, to $1.18\mu s$ for 60 kHz and to 147 ns for 480 kHz. As a result, the cyclic prefix overhead remains constant at about 7% irrespective of sub-carrier spacing. Generally, larger subcarrier spacing reduces sensitivity to frequency dispersion,

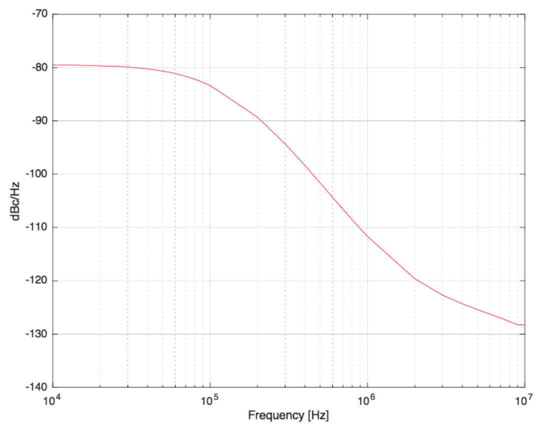

Fig. 2. Phase noise power spectral density of the simulated phase noise spectrum.

at the price of higher sensitivity of the delay dispersion. Due to the larger Doppler spread and the phase noise, it is anticipated that mm-wave systems need larger subcarrier spacing than sub-6GHz systems.

For the channel model, we adopt here the Urban Macrocell (UMa) channel model of 3GPP, described in [9]. In additon, phase noise was modeled according to the model in [10]. The simulated phase noise spectrum is shown in Fig. 2. Additional simulation parameters are described in Table I.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Carrier frequency	28 GHz
Bandwidth	100 MHz
Sub-carrier spacing	15 kHz, 60 kHz, 480 KHz
Frame duration	1ms
Modulation	16-QAM
Coding rate	2/3
Channel code	LTE turbo code
Channel model	3GPP CDL-C
Channel delay spread	266 ns

B. Results

Simulation results for OTFS are shown in Figure 3, and compared to OFDM for the mentioned sub-carrier spacings of 15 kHz, 60 kHz, and 480 kHz. Specifically, we show the coded bit error probability (BER). The 60 kHz spacing provides the best performance (at least for reasonably low coded BERs). The 480 kHz sub-carrier spacing may not be used with this set of parameters, due to insufficient length of the cyclic prefix. Note that, while extending the cyclic prefix might improve BER performance, it would result in unacceptably high overhead and loss of spectral efficiency. Moreover, the performance with 15 kHz sub-carrier spacing is degraded with respect to the 60 kHz sub-carrier spacing case, due to the higher degradation caused by frequency dispersion (both from phase noise and Doppler spread). For OFDM, the degradation is unacceptable, showing a distinct error floor. Similar conclusions were obtained in other simulation settings (not shown here for space reasons). For all cases, OTFS outperforms OFDM in the presence of frequency dispersion, making it a strong candidate for millimeter wave systems.

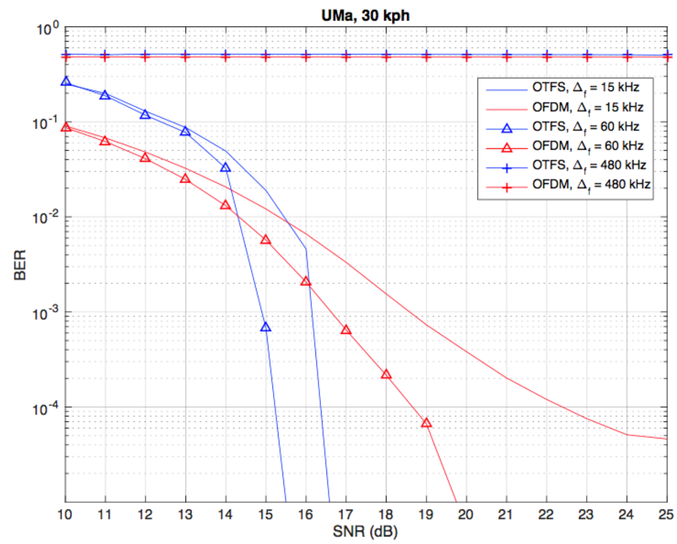

Fig. 3. BER of OTFS for different subcarrier spacings, compared to OFDM.

IV. CONCLUSION

This paper has analyzed the performance of a new modulation format, OTFS, in mm-wave systems, analyzing in particular the impact of phase noise, Doppler spread, and delay spread, which might have different values compared to the traditional cellular bands. It was demonstrated that there are a number of situations in which OTFS outperforms traditional OFDM modulation.

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